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Forecasting risk, informed speculation, and financial innovation

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Abstract

Speculators who prey on hedgers can stifle financial innovation in the sense that new markets can fail. In this paper I analyze whether a profit maximizing exchange nonetheless chooses to open markets for speculative securities and if so, how to circumvent the problem of market failure. I find that the optimal financial innovation takes two forms. The first is a market structure consisting of hedge instruments, traded in low volume at stable asset prices. The second is a market structure consisting of speculative instruments, traded in greater volume at volatile asset prices. These strategies are derived within the same framework where the cost and the quality of the speculators' information set and the hedgers risk aversion ultimately determine which is the optimal one.

JEL classification: G11; G12; G14; G23

Keywords: Financial innovation; Speculation; Trading volume; Volatility

1. Introduction

How do exchanges deal with problems of potential market failure caused by informational asymmetries and what is the role of financial innovation in this? In addressing this question, this paper develops two themes. The first theme is linked to the trading of financial instruments. Hedging—where risk averse traders enter the financial market in order to hedge endowment risk—is one motive for trade. Informed speculation—where traders (not necessarily risk averse) acquire information that can be used to earn profits—is another. These are linked in that spec-

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ulators who enter a market prey on hedgers. When the information asymmetry is large various forms of market failure may arise. The second theme is linked to the financial innovation process which deals with which financial markets are open for trade. This process determines, in effect, the extent to which the trading venue can protect the hedgers from the predatory behavior of speculators. There is quite a large literature on financial innovation (see Duffie and Rahi, 1995 and Allen and Gale, 1994 for useful surveys) but some aspects addressed here are still unexplored. In particular, this paper investigates how an exchange deals with the problem of market failure while seeking to maximize its commission revenue. The exchange makes profits from targeting both the market for hedge instruments and the market for speculative instruments, but because the activity of speculators generally interferes with the activity of hedgers the optimization problem for the exchange is not a trivial one. This paper seeks to find a solution to this program in a relatively simple model, but nonetheless one in which the externalities of speculation on hedge motivated trade are clearly recognized.

The paper is motivated by stylized empirical evidence of a diversity of driving forces behind financial innovation. We observe general assets that tend to be exchange-traded, attracting a broad class of investors trading in large volume. We also observe special instruments, sold mainly over-the-counter to a relatively limited market segment, and attracting a much lower trading volume.[1] Speculators thrive on instruments traded in large volume and should therefore be expected to interfere more with exchange-traded securities. If this is so, why are these markets so successful in avoiding market failure? We address this question by analyzing a simple model in which parameters describing the agents' information set and risk aversion are varied to generate this variety of optimal markets. The quality of the speculators' information set is crucial. Several papers (e.g. Marín and Rahi, 2000 and Dow, 1998) analyze the incentives to trade a new security but fix the information structure. Others (e.g. Grossman and Stiglitz, 1980 and Bhushan, 1991) endogenize the information acquisition process but take the market structure as fixed.[2] This paper provides a link between the two, and show that information structure and market structure are associated. When the information is sufficiently costly and precise the exchange chooses to list hedge securities only. Otherwise, the exchange lists speculative securities.

The fact that the information structure and the market structure are linked suggests that price volatility and trading volume of assets (driven by speculation) is also a function of the markets in which the asset is listed. The analysis demonstrates that the small hedge markets have lower price volatility and lower trading volume than the general speculative markets. The innovation process generates, therefore, two sets of markets. One set contains the markets for insurance instruments which are traded in low volume with stable asset prices. The other set contains the markets for speculative instruments which are traded sometimes in very high volume and have volatile asset prices.

Some of the key ideas here have been put forward elsewhere in slightly different contexts. Marín and Rahi (1999), for instance, show that speculation may be welfare improving, and Dow (1998) shows that listing an index can generate higher welfare than listing its constituents separately. Unlike these papers, the decision to acquire information is endogenized in our model. The profitability of attracting speculative traders to a trading venue is, therefore, measurable within the context of the model. A closely related paper is Hara (1995) which analyzes a similar prob-

[1] Note that some very high volume derivative markets such as the FRA and swap markets are over-the-counter, so the link between trading volume and how trade is organized that is alluded to here is not as clear cut in practice.

[2] There are security design papers in which security design and information acquisition incentives are jointly and endogenously determined (e.g. Back, 1993 and Boot and Thakor, 1993).

lem but uses a framework in which there are no informational asymmetries and, therefore, no speculation.

Section 2 outlines the basic model. Section 3 analyzes the problem of identifying the optimal financial innovation strategy for the exchange, and Section 4 concludes.

2. The model

This section sets out the basic framework. There are two types of agents. The first type is risk averse with a strictly increasing and concave von Neumann–Morgenstern utility function u. We suppose u is a CARA (constant absolute risk aversion) utility function and takes the form

$$u(x) = -\exp(-\gamma x) \tag{1}$$

with risk aversion coefficient $\gamma > 0$. The risk averse agents are infinitesimal, indexed by the interval $[0, 1]$, and each exposed to a perfectly correlated endowment risk, $\widetilde{w}_0$. The second type of agents is risk neutral. These agents, of which there are infinitely many, have access to private information about certain risk factors at a cost $c > 0$ and are not infinitesimal so are able to recognize that their own actions can affect prices. Since the information cost is strictly positive and the gains from speculation are finite, there will be at most a finite subset of risk neutral agent investing in information and participating in trade in any equilibrium.

Next we describe the financial system. There exists a trading venue—an exchange—in which financial assets or securities are traded. All participating in trading (hedging or speculation) must pay a fixed commission fee, denoted T, levied by the exchange. The exchange seeks to list the set of securities enabling it to maximize its total commission revenue. There are four risk factors in the economy, $\tilde{x}$, $\tilde{y}$, $\widetilde{R}$, and $\tilde{s}$. The first three are independent and bivariate $+1/-1$ random variables where the two outcomes are equally likely. These random variables determine all exogenous uncertainty in the economy. Specifically, the financial assets available to the exchange take the form

$$\tilde{z}_i = \tilde{x} + \beta_i \tilde{y} \tag{2}$$

and the endowment risk of the risk-averse individuals takes the form

$$\widetilde{w}_0 = \widetilde{R}\tilde{x}. \tag{3}$$

The random variable $\widetilde{R}$ is private knowledge to the risk-averse agents. When a risk-neutral individual pays the information cost c, he receives a signal $\tilde{s}$ about the random variable y. This signal, which is also a bivariate $+1/-1$ random variable, has the precision level $\rho \in [0, 1]$, defined by the relationship

$$\Pr[\tilde{y} = k \mid \tilde{s} = k] = \frac{1+\rho}{2}. \tag{4}$$

When $\rho = 0$ the signal is uninformative, and when $\rho = 1$ the signal is perfect. Given that the market for an asset is open the incoming orders are cleared by a market maker similarly to Kyle (1985). The market makers are risk-neutral agents who make zero expected profits conditional on the information that can be inferred from the order flow. We assume that each individual market maker is restricted to observe the order flow in his market only, i.e. that it is not possible to observe the order flow across two or more markets before setting prices.

The end-of-period wealth is

$$\widetilde{w}_H = \widetilde{w}_0 + \sum_i t_H^i(\tilde{z}_i - \tilde{p}_i) - T \tag{5}$$

for the agents who are risk averse, and

$$\widetilde{w}_S = -c + \sum_i t_S^i(\tilde{z}_i - \tilde{p}_i) - T \tag{6}$$

for those who are risk neutral. Here, t_H^i denotes the order by a risk-averse trader (hedger) in the market for security $\tilde{z}_i$ and t_S^i denotes the corresponding order by a risk-neutral trader (speculator) in the same market. The price $\tilde{p}_i$ is the price set by the market maker for the market of security $\tilde{z}_i$. The market maker in the market for security $\tilde{z}_i$ makes the price $\tilde{p}_i$ conditional on the aggregate order flow in this market, t^i, where

$$t^i = t_H^i + \phi t_S^i \tag{7}$$

where ϕ is the number of the subset of the set of risk neutral agents who become informed. All the informed agents receive the same signal, which may be correct or incorrect where the likelihood of error depends on the precision level of the signal.

Before we proceed we list a few comments on the model specification above. The first is related to the specification of fundamental risk. The model separates risk factors into a random variable associated with hedging endowment shocks and an independent random variable associated with speculation. One might wonder why this choice is made, especially as the literature largely does not make this distinction. The primary reason is that we wish to investigate the incentive for pooling speculative and hedge-related risk in a setting where assets are formed endogenously. Using the same random variable as the basis for both hedge driven trade and speculation we exogenously impose a link between hedging and speculation that makes it difficult to analyze the issue of interest in this paper.

Next, we model the trade commission as a fixed cost. In practice such commissions tend to be a mixture of fixed and variable costs, e.g. a fixed trading commission and a bid–ask spread. Allen and Gale (1994) discuss this problem in Chapter 8 of their book and analyzes the efficiency properties of a model in which exchanges can charge lump sum fees to cover their setup cost. Hara (1995), in contrast, assumes that the commission has no fixed cost component. The fixed cost assumption is made here for two key reasons. Tractability is an important factor as a fixed cost gives us a framework in which we are able to focus on the security design issue without the interference brought about by a proportional cost element of trading. Also—and this point is related to the first—a fixed cost is optimal within the framework chosen (see Allen and Gale's, 1994). Fixed costs lead to inefficiencies through exclusion of agents (typically, those with low trading motive) whereas variable costs lead to inefficiencies through limiting the trading activities of the agents who do choose to enter the market. Here, since all hedgers are homogeneous and all speculators earn zero rent in ex ante terms, the costs of exclusion are zero.

Our model assumes that the market makers are prevented from observing the aggregate order flow in all but their own markets. If two linearly independent securities are listed in our model, the aggregate order flows reveal perfectly the speculative risk factor $\tilde{y}$. This follows from the primitive specification of risk (i.e. that risk factors are a simple bivariate random variable) and would not be true with a more realistic probabilistic structure. To avoid information revelation arising primarily on account of cross-market observations, therefore, we put restrictions on the market makers' information sets.

The model also assumes the risk-averse agents are infinitesimal while the risk-neutral agents are not. This assumption is added in order to avoid existence problems of equilibrium. The risk-neutral agents acquire information ex ante in order to participate in speculative trading ex post. If we assume an equilibrium exists in which speculators acquire information ex ante it must be the case that the expected trading profits ex post at least cover the information cost. This necessitates both that speculative assets are listed and that the speculators who trade do not trade too much in order to prevent fully revealing prices for sure ex post. But clearly this is not possible if each agent is a price taker since he then has an incentive to exploit his informational advantage to the fullest extent. On the other hand, if we assume an equilibrium exists in which no agent acquires information ex ante there exist unexploited speculative profits ex post when speculative assets are listed. But this is also clearly not possible since some agents would have an incentive to buy information ex ante. To solve this type of existence problem we need to assume that the risk-neutral agents are not infinitesimal. We shall, however, make these agents sufficiently small to avoid problems associated with divisibility. With some abuse of notation, therefore, the paper proceeds talking about a continuous finite "measure" of speculators although we really have in mind a finite number.

Finally, the model assumes that speculators receive perfectly correlated signals even though the signal is not necessarily perfect. This specification is chosen because of problems related to the law of large numbers. Even when speculators receive independent signals the noise in the aggregate speculative order flow nonetheless goes to zero if sufficiently many speculators trade. That the market maker infers the asset payoffs better than any one speculator with private information is of course not unreasonable in itself, however, it does appear unrealistic given our existing knowledge of how markets work in practice. Additionally, as is demonstrated later on, this case leads to non-existence of a trading equilibrium.

3. The optimal asset structure

This section seeks to derive the optimal asset structure which will be given as the solution to a maximization program for the exchange. This program specifies both the process of choosing which markets to open and how much to charge for access to these markets. The programme can be stated as

$$\max_{(\beta_1,\beta_2),T}(1+\phi)T. \tag{8}$$

The exchange seeks to list an asset structure consisting of at most two assets (β_1, β_2) and charge a commission per trader of T such that its total revenue, i.e. the total measure of hedgers and speculators times the commission T, is maximized. The optimization program is subject to a participation constraint for the hedgers

$$Eu(\widetilde{w}_H) \geqslant Eu(\widetilde{w}_0) \tag{9}$$

and a non-negative expected profit constraints for the speculators

$$E(\widetilde{w}_S \mid \phi) \geqslant 0. \tag{10}$$

The inequality (10) may even in equilibrium be strict since the speculators are not infinitesimal. However, if we adjust the size of each agent, e.g. by making them arbitrarily small, (10) must hold with equality in any equilibrium. The remainder of the analysis supposes that this is the case.

The program in (8) is also subject to the condition that all markets are cleared by a market maker who sets prices in a competitive manner. The prices are, therefore, given by

$$p_i = E\left(\tilde{x} + \beta_i \tilde{y} \mid t^i\right). \tag{11}$$

We restrict the asset structures to two assets described by the parameters (β_1, β_2). Now, it is true that when there are only two risk factors a third asset is redundant in a spanning sense. However, Hara (1995) demonstrates that redundant assets can play a role because span is not the only issue here. When the commission revenue is proportional to trading volume, the commission paid by the individual hedger becomes a function of the particular insurance portfolio held. In some cases the exchange may find it beneficial to sell certain portfolios at a discount, which can be achieved by listing a redundant asset. In other words, a redundant asset makes it possible to obtain a degree of price discrimination in the market for insurance portfolios. In our model price discrimination is never an issue because the hedgers are identical. With a fixed fee commission and a homogeneous group of hedgers the marginal cost of trading (conditional on the fact that it is optimal to trade at all) is always zero such that the hedge portfolios are economically efficient even if the exchange is a monopolist. Our model isolates, therefore, the interactive effects between hedging and speculation without including further interfering effects created by a volume-dependent transaction cost. Within our framework, consequently, the restriction to maximally two assets leads to no loss of generality.

The first step is to analyze the trading behavior of the traders who actively trade on the exchange. The hedgers' optimization program is stated as

$$\max_{t_H^1, t_H^2} Eu(\widetilde{w}_H) \tag{12}$$

subject to

$$\widetilde{w}_H = \widetilde{w}_0 + t_H^1(\tilde{z}_1 - \tilde{p}_1) + t_H^2(\tilde{z}_2 - \tilde{p}_2). \tag{13}$$

In order to obtain the optimal hedge demand functions we need to take into account price formation in the market maker markets. The following lemma establishes a price formation process consistent with the zero profit condition for the market makers. Before we proceed, however, we need to make assumptions describing the trading strategy of the informed speculators. The informed speculators are assumed to adopt *symmetric trading strategies* where they are buying as many units on good information as they are selling on bad information in all open markets. This assumption is firstly a very natural one in an otherwise symmetric model. Secondly, it is not really a restrictive one. We could envisage the speculators were to trade asymmetrically, e.g. buy more aggressively on good information than they sell on bad, in which case the market maker would expect to go short in the stock to make up for the disproportionally (in expectation) aggressive speculative buying pressure. The speculative gains to the speculators and the utility of the hedgers would, however, be exactly the same as it would be under symmetric trading. What would change is the market maker's price setting rule, but this change absorbs on the other hand all the effects from asymmetric trading. Lemma 1 gives, therefore, the price setting rule for the symmetric case only.

Lemma 1. *Suppose the speculators adopt symmetric trading strategies. If $\phi = 0$ then $p_i = 0$. Otherwise, the price-setting rule,*

$$p_i = E\left[\tilde{z}_i \mid t^i\right] = \begin{cases} \rho\beta_i & \text{if } t^i > 0, \\ 0 & \text{if } t^i = 0, \\ -\rho\beta_i & \text{if } t^i < 0, \end{cases} \tag{14}$$

is the only one that in equilibrium is such that the asset prices equal the expected asset payoffs conditional on the aggregate order flows.

Lemma 1 illustrates the convenience of the chosen uncertainty structure. Prices are either completely non-informative or fully revealing. When there are no speculators entering the market, the prices must always be non-informative by the zero-profit condition of the market maker. If speculators are entering the market, the price becomes very sensitive to asymmetric aggregate demand, i.e. the market becomes very "shallow." This is necessary as a device to discipline the speculators' trading activities. If the market maker allows "depth" in the market it would always be optimal for some speculator (particularly if small) to deviate from a given strategy push the aggregate demand away from zero. It is, therefore, only when the aggregate demand is zero in equilibrium that the price is also set at zero and the price responds infinitely quickly to small buying and selling pressure at this point.

Lemma 1 restricts, in effect, the search of equilibria in which speculators operate to those in which prices are non-informative if and only if the aggregate order flow is zero. The following result, based on Lemma 1, derives the optimal demand schedules for the hedgers. Here it is assumed by convention that asset 2 is always the most speculative asset, i.e. that $\beta_2 \geqslant \beta_1$. This convention is kept throughout the remaining parts of the paper.

Proposition 1. *The optimal demand schedules for the hedgers are given by two functions $t_H^1(\beta_1, \beta_2, \rho)$ and $t_H^2(\beta_1, \beta_2, \rho)$ satisfying the condition*

$$\begin{aligned}&\left(1+\beta_1\left(1-\frac{\rho(1-\rho)}{2}\right)\right)t_H^1+\left(1+\beta_2\left(1-\frac{\rho(1+\rho)}{2}\right)\right)t_H^2\\&\quad=1+\ln\left(\frac{\beta_2-\beta_1+\rho\beta_1\beta_2}{\beta_2-\beta_1-\rho\beta_1\beta_2}\right)^{\frac{1}{2\gamma}}.\end{aligned}\tag{15}$$

This relationship is defined only if $\beta_2 > \beta_1/(1-\rho\beta_1) \geqslant 0$.

Proposition 1 highlights the restrictions imposed on the demand functions of the hedgers by the equilibrium conditions. It remains to identify these functions; this is the subject for the next result. Crucial here are the issues of existence and uniqueness. Hara (1995) demonstrates problems with existence in his framework, and it will be shown that this is also a problem in our framework for certain parameters. Where existence can be demonstrated, however, we show that uniqueness obtains. The optimal order flows, i.e. the functions that satisfy Eq. (15) together with the first-order conditions for expected utility maximization, are given below.

Proposition 2. *The optimal order flows satisfying the relationship in (15) are given by*

$$|t_H^1|=\frac{\beta_2(1-\rho(1+\rho)/2)}{\beta_2(1-\rho(1+\rho)/2)-\beta_1(1-\rho(1-\rho)/2)}\left(1+\ln\left(\frac{\beta_2-\beta_1+\rho\beta_1\beta_2}{\beta_2-\beta_1-\rho\beta_1\beta_2}\right)^{\frac{1}{2\gamma}}\right),\tag{16}$$

$$|t_H^2|=\frac{\beta_1(1-\rho(1-\rho)/2)}{\beta_2(1-\rho(1+\rho)/2)-\beta_1(1-\rho(1-\rho)/2)}\left(1+\ln\left(\frac{\beta_2-\beta_1+\rho\beta_1\beta_2}{\beta_2-\beta_1-\rho\beta_1\beta_2}\right)^{\frac{1}{2\gamma}}\right)\tag{17}$$

where $\operatorname{sign}(t_H^1)=-\operatorname{sign}(t_H^2)$. These are well defined only if $\rho<1$, $\beta_1<\frac{1}{\rho}$, and finally

$$\beta_2>\max\left(\frac{\beta_1}{1-\rho\beta_1},\frac{1-\rho(1-\rho)/2}{1-\rho(1+\rho)/2}\beta_1\right).$$

We are unable to define functions describing the hedge demand for the case where the speculators have perfect information (i.e. $\rho = 1$). The sum $t_H^1 + t_H^2$ equals the right-hand side in (15) so the functional forms are ultimately defined by the relationship

$$\beta_1\left(1 - \frac{\rho(1-\rho)}{2}\right)t_H^1 + \beta_2\left(1 - \frac{\rho(1+\rho)}{2}\right)t_H^2 = 0.$$

At the point of perfect information, the coefficient on t_H^2, $\beta_2(1 - \rho(1+\rho)/2)$, vanishes. The only feasible candidate is, therefore,

$$t_H^1 = 0 \quad \text{and} \quad t_H^2 = 1 + \ln\left(\frac{\beta_2 - \beta_1 + \rho\beta_1\beta_2}{\beta_2 - \beta_1 - \rho\beta_1\beta_2}\right)^{\frac{1}{2\gamma}},$$

but in this case the agents trade in only one market at the same time as submitting orders which depend partially on the non-traded asset. This is, of course, nonsense, so for this special case there exists no optimal order flows satisfying the first-order conditions. Existence is also restricted when the information is imprecise. One asset has an upper bound on the relative loading on the speculative risk factor and the other asset has a lower bound on this loading. The reason these bounds come into play is to prevent the assets becoming too similar. When that happens, the hedgers are forced to trade in too high volume, typically by taking a large long position in one asset and an offsetting large short position in the other in order to generate the desired hedge effect. However, since higher volume is associated with higher adverse selection costs, there exists no optimal hedge demand when the assets are too similar. This is also the reason why the bounds weaken as the precision of the information signal is reduced.

We return to the problem of deriving the level of speculation in the market. Each speculator expects trading profit $E(\widetilde{w}_S \mid \phi)$, which is a function of the mass of the speculators, ϕ. The speculators, therefore, need to make their trades conditional on the number of speculators entering the market. It is also apparent that the more the hedgers trade (in absolute terms) the more each speculator is able to trade since the speculators' profit is increasing in the hedge motivated trading volume. The more the hedgers trade, therefore, the more speculators enter the market. The expected speculative gains for each speculator can, consequently, be derived as follows:

$$\begin{aligned} E(\widetilde{w}_S) = {} & \frac{1+\rho}{2}\left(\frac{1}{2}\left(\left|t_S^1\right|\beta_1(1-\rho) + \left|t_S^2\right|\beta_2\right) + \frac{1}{2}\left(\left|t_S^1\right|\beta_1 + \left|t_S^2\right|\beta_2(1-\rho)\right)\right) \\ & + \frac{1-\rho}{2}\left(\frac{1}{2}\left(\left|t_S^1\right|\beta_1(-1-\rho) + \left|t_S^2\right|\beta_2(-1)\right)\right. \\ & \left. + \frac{1}{2}\left(\left|t_S^1\right|\beta_1(-1) + \left|t_S^2\right|\beta_2(-1-\rho)\right)\right) - T - c \qquad (18) \\ = {} & \frac{\rho}{2}\left|t_S^1\right|\beta_1 + \frac{\rho}{2}\left|t_S^2\right|\beta_2 - T - c. \qquad (19) \end{aligned}$$

Taking into account the constraints on speculation given by Eqs. (16), (17) and the zero-profit condition by Eq. (10), we obtain the following result which also establishes the optimal strategy for setting commission fees for the traders.

Proposition 3. *The trading commission T is determined* (*given β_1 and β_2*) *at the point where the participation constraint for hedgers is binding. When the non-negative profit condition for the speculators hold with equality, the mass of speculative traders entering the market is then given* *by the function $\phi(T)$, where*

$$\phi(T)=\frac{\beta_1\beta_2\rho\left(1-\frac{\rho}{2}\right)}{(T+c)\left(\beta_2\left(1-\frac{\rho(1+\rho)}{2}\right)-\beta_1\left(1-\frac{\rho(1-\rho)}{2}\right)\right)}\left(1+\ln\left(\frac{\beta_2-\beta_1+\rho\beta_1\beta_2}{\beta_2-\beta_1-\rho\beta_1\beta_2}\right)^{\frac{1}{2\gamma}}\right). \tag{20}$$

The next step is to work out an exact expression for the trade commission T. Using the fact that the participation constraint for the hedgers is always binding this is straightforward. Substituting the optimal market orders given in Proposition 2 into the table showing state contingent wealth (given in Appendix A), we find an expression for the expected utility in which the trade commission T can be isolated:

$$E\left(u(\widetilde{w}_H)\right)=-\exp(\gamma T)E\left(\exp\left(\tilde{A}(\beta_1,\beta_2,\rho)\right)\right). \tag{21}$$

We detail in Proposition 4 the form of the function $\tilde{A}$. At the point where the hedgers achieve a perfect hedge without the interference of speculation the term $\exp(A)=1$ so the trade commission is given by the number T defined by

$$E\left(u(\widetilde{w}_H)\right)=-\exp(\gamma T)=-\frac{\exp(\gamma)+\exp(-\gamma)}{2}=E\left(u(\widetilde{w}_0)\right)$$

where the right-hand side is the no-trading expected utility (i.e. the outside option for the hedgers). Therefore, in this particular case,

$$T=\frac{1}{\gamma}\ln\left(\cosh(\gamma)\right)$$

and this is also the maximum trade commission the exchange can levy on the traders who seek to hedge their endowment risk.

Proposition 4. *The profit-maximizing trade commission, given an asset structure for which a trading equilibrium exists, is*

$$T=\frac{1}{\gamma}\ln\left(\cosh(\gamma)\right)-\frac{1}{\gamma}\ln\left(E\exp\left(\tilde{A}(\beta_1,\beta_2;\gamma,\rho)\right)\right) \tag{22}$$

where the function $\tilde{A}$ is given by

$$\begin{aligned} A={}&\pm\ln\left(\frac{\beta_2-\beta_1+\rho\beta_1\beta_2}{\beta_2-\beta_1-\rho\beta_1\beta_2}\right)^{\frac{1}{2\gamma}}\\ &\pm\frac{\left(1+\ln\left(\frac{\beta_2-\beta_1+\rho\beta_1\beta_2}{\beta_2-\beta_1-\rho\beta_1\beta_2}\right)^{\frac{1}{2\gamma}}\right)\rho\beta_1\beta_2}{\beta_2\left(1-\frac{\rho(1+\rho)}{2}\right)-\beta_1\left(1-\frac{\rho(1-\rho)}{2}\right)}\begin{cases}\left(1+\frac{\rho(1-\rho)}{2}\right), & \textit{if signal } s=1,\\ \left(1-\frac{\rho(3-\rho)}{2}\right), & \textit{if signal } s=-1,\end{cases}\end{aligned} \tag{23}$$

where the first sign is positive if and only if $x=1$ and the next sign is negative if and only if the information signal of the speculators is true.

We now have sufficient data to start the main part of the analysis which deals with finding the optimal financial innovation program for the exchange. Recalling the objective function in (8), we can now eliminate the variables T and $\phi(T)$ as these are given as functions of the variables γ, ρ, β_1, and β_2. We find, moreover, that we can rewrite the objective function as a maximization program over the asset structure (β_1, β_2) only:

$$\max_{(\beta_1,\beta_2)} \big(1+\phi(T)\big)T \tag{24}$$

where $\phi(T)$ is given by Proposition 3 and T is given by Proposition 4. Substituting for these, we find the explicit functional form of the objective function. A problem persists in operationalizing the maximization program, and a particular obstacle is the term $\ln(E\exp(\tilde{A}))$ from (22). It is useful in this respect to transform the objective function using the hyperbolic sine and cosine functions to express this term. Some straightforward but lengthy transformations derives the following expression from (24) which enables us to proceed.

$$\begin{aligned}\max_{\beta_1,\beta_2} \Pi = {} & \ln\left(\frac{\cosh(\gamma)}{\frac{1}{4}\sum_i \cosh(i) - \rho\sinh(i)}\right)^{\frac{1}{\gamma}} \\ & + \frac{\ln\left(\frac{\cosh(\gamma)}{\frac{1}{4}\sum_i \cosh(i)-\rho\sinh(i)}\right)^{\frac{1}{\gamma}}}{c+\ln\left(\frac{\cosh(\gamma)}{\frac{1}{4}\sum_i \cosh(i)-\rho\sinh(i)}\right)^{\frac{1}{\gamma}}}\rho\left(1-\frac{\rho}{2}\right)\kappa\big(1+\ln\delta^{\frac{1}{2\gamma}}\big)\end{aligned} \tag{25}$$

where

$$\delta = \frac{\beta_2-\beta_1+\rho\beta_1\beta_2}{\beta_2-\beta_1-\rho\beta_1\beta_2}, \tag{26}$$

$$\kappa = \frac{\beta_1\beta_2}{\beta_2(1-\rho(1+\rho)/2)-\beta_1(1-\rho(1-\rho)/2)}, \tag{27}$$

and $i = a, b, c, d$, where

$$a = \frac{\ln\delta}{2\gamma}\big(1-\kappa\rho\big(1+\rho(1-\rho)/2\big)\big)-\kappa\rho\big(1+\rho(1-\rho)/2\big), \tag{28}$$

$$b = \frac{\ln\delta}{2\gamma}\big(1-\kappa\rho\big(1-\rho(3-\rho)/2\big)\big)-\kappa\rho\big(1-\rho(3-\rho)/2\big), \tag{29}$$

$$c = \frac{\ln\delta}{2\gamma}\big(-1-\kappa\rho\big(1+\rho(1-\rho)/2\big)\big)-\kappa\rho\big(1+\rho(1-\rho)/2\big), \tag{30}$$

$$d = \frac{\ln\delta}{2\gamma}\big(-1-\kappa\rho\big(1-\rho(3-\rho)/2\big)\big)-\kappa\rho\big(1-\rho(3-\rho)/2\big). \tag{31}$$

At the corner $\beta_1 = 0$ the exchange lists a single instrument that depends on the hedge risk $(\tilde{x})$ only. At this corner, moreover, we find that the transformed variables δ and κ take on the values 1 and 0, respectively. Substituting these values into the system (25)–(31) fixes the profit of the exchange to the number $\ln(\cosh(\gamma))^{1/\gamma}$, which equals the trade commission levied on the unit mass of hedgers when their participation constraint is just binding. The profit goes to zero for $\gamma \to 0$, confirming the intuition that at the point at which the hedgers become risk neutral the exchange makes no profits by opening markets for insurance instruments. The corner solution at $\beta_1 = 0$ is particularly interesting as it can be shown to be optimal for a certain range of parameters. For all remaining cases where a trading equilibrium exist, we find that an interior solution, involving listing speculative securities, is optimal. The specific conditions are given below.

Proposition 5. *The corner solution $\delta = 1$ and $\kappa = 0$ is suboptimal provided the information cost is not too great, in particular,*

$$\rho c < \ln\bigl(\cosh(\gamma)\bigr) \tag{32}$$

and that the hedgers are sufficiently risk averse, in particular,

$$\gamma > \frac{\ln(\cosh(\gamma))}{\ln(\cosh(\gamma)) - \rho c}. \tag{33}$$

In this case there always exists an optimal interior solution where the exchange lists speculative securities only. If the bounds in (32) or (33) are violated, the exchange always picks the corner solution $\beta_1 = 0$ (where $\delta = 1$ and $\kappa = 0$) and lists a single hedge instrument which depends on $\tilde{x}$ risk only.

The inequality in (32) has a fairly straightforward interpretation. The right-hand side is, in effect, a measure of the strength of the hedging motive. It takes value zero when the hedgers are risk neutral, and is increasing in the risk aversion coefficient γ. If the adverse selection cost is large relative to the strength of the hedging motive the inequality cannot be satisfied, as the hedgers withdraw from trade too quickly. The left-hand side is the product of the precision parameter and the information cost. The left-hand side is partly measuring the adverse selection problem caused by informed speculation. As the precision parameter ρ increases, the informational advantage of the speculators increases. That speculators appropriate rent from the hedgers is obviously harmful to the exchange, which then must reduce the commission levied on the hedgers to maintain their participation constraint. The left-hand side also measures the intensity of the influx of speculators. As the information cost c increases, the speculators must make up for the higher information cost by making bigger speculative profits. This means fewer speculators arrive, and this is also harmful to the exchange as the commission raised from speculation is reduced. Cheap and imprecise information coupled with a strong non-informative hedge motive are, therefore, associated with speculative markets.

Also, we find that the risk aversion coefficient must be greater than

$$\frac{\ln(\cosh(\gamma))}{\ln(\cosh(\gamma)) - \rho c} > 1$$

in order that the exchange lists speculative securities (Eq. (33)). The intuition for this is also relatively straightforward. Unless the hedgers are sufficiently risk averse, the loss of trade commission necessary to maintain the hedgers participation constraint is never compensated by the extra commission revenue generated by the influx of speculators. Risk aversion does, therefore, play a certain role in speculative markets.

The main message is, however, that the financial innovation process links certain information structures, defined by the price and quality of private information available, with certain market structures. Taking this further, we should also expect that the price volatility is a function of market structure. Since speculators are discouraged to operate in certain markets but encouraged in others, we expect the degree of information revelation in prices to vary also. A specialist insurance market where the instrument traded does not depend on speculative risk has zero price volatility and unit payoff volatility (by normalization). A market that is part of an asset structure that attracts speculation has, in contrast, strictly positive price volatility. The price variance (using Lemma 1) is given by

$$\mathrm{Var}(\tilde{p}_i) = \frac{\rho^2}{2}\beta_i^2 > 0 \tag{34}$$

whereas the payoff variance is given by

$$\mathrm{Var}(\tilde{z}_i) = 1 + \beta_i^2. \tag{35}$$

Consequently, a speculative market has higher price volatility as $\rho\beta_i/\sqrt{2} > 0$, it has higher payoff volatility as $\sqrt{1+\beta_i^2} > 1$, and higher relative price-to-payoff volatility as $\rho\beta_i/\sqrt{2(1+\beta_i^2)} > 0$. The "extra" price volatility associated with speculative markets is, however, not akin to excess volatility as defined by Shiller (1981). The price-to-payoff volatility is always less than one.

Also, we expect that market structure and trading volume are linked. The trading volume in a specialist insurance market is (by normalization) equal to 1. The trading volume in a speculative market equals

$$|t_H^i| + \phi|t_S^i| = 2|t_H^i| \tag{36}$$

and we shall demonstrate that for at least one market this is always greater than twice the trading volume in a pure hedge market. To see this, suppose $\ln \delta^{1/(2\gamma)} < 1$, in which case

$$|t_H^1| + |t_H^2| > \frac{\beta_2\left(1 - \frac{\rho(1+\rho)}{2}\right) + \beta_1\left(1 - \frac{\rho(1-\rho)}{2}\right)}{\beta_2\left(1 - \frac{\rho(1+\rho)}{2}\right) - \beta_1\left(1 - \frac{\rho(1-\rho)}{2}\right)} \tag{37}$$

by the fact that $\delta > 1$. Subtracting $|t_H^2|$ from both sides, we obtain the inequality

$$|t_H^1| > 1 + \left(1 - \ln \delta^{\frac{1}{2\gamma}}\right) \frac{\beta_1\left(1 - \frac{\rho(1-\rho)}{2}\right)}{\beta_2\left(1 - \frac{\rho(1+\rho)}{2}\right) - \beta_1\left(1 - \frac{\rho(1-\rho)}{2}\right)} \tag{38}$$

which is greater than 1 by assumption. Now suppose $\ln \delta^{1/(2\gamma)} \geqslant 1$, which implies that

$$|t_H^1| + |t_H^2| \geqslant 2\frac{\beta_2\left(1 - \frac{\rho(1+\rho)}{2}\right) + \beta_1\left(1 - \frac{\rho(1-\rho)}{2}\right)}{\beta_2\left(1 - \frac{\rho(1+\rho)}{2}\right) - \beta_1\left(1 - \frac{\rho(1-\rho)}{2}\right)} > 2 \tag{39}$$

so also in this case $|t_H^1|$ or $|t_H^2|$ is greater than 1.

4. Concluding remarks

This paper seeks to understand how an exchange applies financial innovation to attract the optimal number of speculators to its markets. The existing literature points to the fact that attracting too many can be harmful since speculators prey on the existing uninformed traders who seek insurance for preference shocks. The paper demonstrates that some speculation can be desirable as it generates extra profits for the exchange and, in particular, the exchange is keen to attract speculators who have access to cheap and imprecise private information when the hedgers are relatively risk averse. The paper establishes, therefore, a link between the information structure of traders and the market structure in which they trade. Where the information costs too much and the asymmetry between the informed and uninformed traders is too big, the financial innovation process separates those who trade to buy insurance and those who speculate. In the converse case the financial innovation process brings them together in the same trading venue where they both trade securities that are speculative.

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Appendix A. Proofs

Proof of Lemma 1. We consider only a single market, hence dropping the index i. When $\phi = 0$ the order flow is always non-informative so the price must be zero. Consider $\phi > 0$. The speculators make their orders contingent on their information and it is straightforward to narrow the optimal strategy down to the one in which they buy the asset given one signal and sell it given the other. The amount traded by the hedgers is a number $+a$ or $-a$ by the symmetry of the model. The amount traded by the speculators is $+b_1$ or $-b_2$ (depending on their signal), where $a, b_1, b_2 > 0$. The aggregate order flow is, therefore, a number

$$t \in \{-a + \phi b_1, a + \phi b_1, a - \phi b_2, -a - \phi b_2\}. \tag{A.1}$$

This number is fully revealing if the values are distinct, but in this case speculation is inconsistent with equilibrium. Obviously, $a + \phi b_1 > 0$ and $-a - \phi b_2 < 0$ can never be equal so we are left with the possibility that

$$-a + \phi b_1 = a - \phi b_2. \tag{A.2}$$

Since the speculators trade the same number of units in both directions,

$$-a + \phi b_1 = a - \phi b_2 = 0 \tag{A.3}$$

which proves the lemma. □

The following showing the state-contingent wealth for the hedgers is useful for the proofs that follow. Table A.1 is derived using Lemma 1.

The table assumes $\phi > 0$ and that the price formation depends on the direction of the hedge-motivated demand: when the hedgers sell asset $\tilde{z}_i$ (where i equals 1 or 2) the price equals $-\rho\beta_i$ when $\tilde{y} = -1$ and 0 otherwise; and when the hedgers buy asset $\tilde{z}_i$ the price equals $\rho\beta_i$ when

Table A.1

Prob	$\tilde{x}$	$\tilde{y}$	$\tilde{s}$	Wealth
$\frac{1+\rho}{8}$	1	1	1	$-1 - T + t_H^i(1 + \beta_i(1-\rho)) + t_H^j(1 + \beta_j)$
$\frac{1+\rho}{8}$	1	−1	−1	$-1 - T + t_H^i(1 - \beta_i) + t_H^j(1 - \beta_j(1-\rho))$
$\frac{1+\rho}{8}$	−1	1	1	$1 - T + t_H^i(-1 + \beta_i(1-\rho)) + t_H^j(-1 + \beta_j)$
$\frac{1+\rho}{8}$	−1	−1	−1	$1 - T + t_H^i(-1 - \beta_i) + t_H^j(-1 - \beta_j(1-\rho))$
$\frac{1-\rho}{8}$	1	1	−1	$-1 - T + t_H^i(1 + \beta_i) + t_H^j(1 + \beta_j(1-\rho))$
$\frac{1-\rho}{8}$	1	−1	1	$-1 - T + t_H^i(1 - \beta_i(1-\rho)) + t_H^j(1 - \beta_j)$
$\frac{1-\rho}{8}$	−1	1	−1	$1 - T + t_H^i(-1 + \beta_i) + t_H^j(-1 + \beta_j(1-\rho))$
$\frac{1-\rho}{8}$	−1	−1	1	$1 - T + t_H^i(-1 - \beta_i(1-\rho)) + t_H^j(-1 - \beta_j)$

$\tilde{y} = 1$ and 0 otherwise. Furthermore, the table also assumes (without loss of generality) that the hedgers are exposed to a negative endowment shock—$\widetilde{w}_0 = -\tilde{x}$. Moreover, it is assumed that they buy $\tilde{z}_i$ and sell $\tilde{z}_j$. The case that $\widetilde{w}_0 = \tilde{x}$ implies the same conditions on the first order conditions for utility maximization for the hedgers given that they sell $\tilde{z}_i$ and buy $\tilde{z}_j$. Although not demonstrated here, it is straightforward to show that the hedgers would never buy both assets or sell both assets at the same time.

Proof of Proposition 1. Here we use the transformation that $\ln \tau_H^i = t_H^i$, and the first order conditions are derived differentiation with respect to τ_H^i rather than t_H^i. There are two first order conditions:

$$\frac{\partial}{\partial \tau_H^1} Eu(\widetilde{w}_H) = 0, \tag{A.4}$$

$$\frac{\partial}{\partial \tau_H^2} Eu(\widetilde{w}_H) = 0. \tag{A.5}$$

Using the table above (which assumes that $t_H^1 > 0$ and $t_H^2 < 0$) we find that these imply the following system

$$A + (B + \rho C_2)\beta_1 + \rho(C_1 - C_2)\beta_1 = 0, \tag{A.6}$$

$$A + (B + \rho C_2)\beta_2 = 0, \tag{A.7}$$

where

$$\begin{aligned} A = {} & \exp(\gamma)\big(1 + x^{-\rho} y^{\rho}\big)\big((1+\rho)x + (1-\rho)y\big) \\ & - \exp(-\gamma)\big(1 + x^{\rho} y^{-\rho}\big)\big((1+\rho)y^{-1} + (1-\rho)x^{-1}\big), \end{aligned} \tag{A.8}$$

$$\begin{aligned} B = {} & \exp(\gamma)\big(1 - x^{-\rho} y^{\rho}\big)\big((1+\rho)x + (1-\rho)y\big) \\ & - \exp(-\gamma)\big(1 - x^{\rho} y^{-\rho}\big)\big((1+\rho)y^{-1} + (1-\rho)x^{-1}\big), \end{aligned} \tag{A.9}$$

$$\begin{aligned} C_1 - C_2 = {} & -\exp(\gamma)\big(1 + x^{-\rho} y^{\rho}\big)\big((1+\rho)x - (1-\rho)y\big) \\ & - \exp(-\gamma)\big(1 + x^{\rho} y^{-\rho}\big)\big((1+\rho)y^{-1} - (1-\rho)x^{-1}\big), \end{aligned} \tag{A.10}$$

$$\begin{aligned} C_2 = {} & \exp(\gamma)\big((1+\rho)x^{1-\rho} y^{\rho} - (1-\rho)y\big) \\ & + \exp(-\gamma)\big((1+\rho)y^{-1} - (1-\rho)x^{-1+\rho} y^{-\rho}\big), \end{aligned} \tag{A.11}$$

where the x and y variables are the following transformations of the hedge demands:

$$x = \big(\tau_H^1\big)^{-\gamma(1+\beta_1(1-\rho))}\big(\tau_H^2\big)^{-\gamma(1+\beta_2)}, \tag{A.12}$$

$$y = \big(\tau_H^1\big)^{-\gamma(1+\beta_1)}\big(\tau_H^2\big)^{-\gamma(1+\beta_2(1-\rho))}. \tag{A.13}$$

Using the fact that the second first order condition implies

$$B + \rho C_2 = -\frac{A}{\beta_2}, \tag{A.14}$$

we can substitute the second equation into the first, and obtain the optimality condition

$$A = -\rho \frac{\beta_1 \beta_2}{\beta_2 - \beta_1}(C_1 - C_2). \tag{A.15}$$

Using the definition of A and $C_1 - C_2$ above, we find that the first order conditions can be reduced to

$$\exp(-2\gamma)x^{-(1-\rho)}y^{-(1+\rho)} = \frac{\beta_2 - \beta_1 + \rho\beta_1\beta_2}{\beta_2 - \beta_1 - \rho\beta_1\beta_2}. \tag{A.16}$$

It is clear that any solution to (x, y) must be in R^2_{++}, so the left-hand side is always positive. A solution will never exist, therefore, if the right-hand side is negative. This implies non-existence of an optimal demand schedules for the hedgers if the right-hand side is zero or negative. Hence

$$\beta_2 > \frac{\beta_1}{1 - \rho\beta_1} \geqslant 0. \tag{A.17}$$

Returning to the problem of identifying the optimal hedge demands, we substitute (A.12) and (A.13) into (A.16) to derive the relationship

$$\left(\tau_H^1\right)^{2\gamma(1+\beta_1(1-\frac{\rho(1-\rho)}{2}))} + \left(\tau_H^2\right)^{2\gamma(1+\beta_2(1-\frac{\rho(1+\rho)}{2}))} = \exp(2\gamma)\frac{\beta_2 - \beta_1 + \rho\beta_1\beta_2}{\beta_2 - \beta_1 - \rho\beta_1\beta_2} \tag{A.18}$$

which implies, taking logarithms on both sides, Eq. (15). □

Proof of Proposition 2. We need to show that if t_H^i, $i = 1, 2$ satisfy the set of first order conditions then no other solution exists. Any other solution f^1 and f^2 can, however, be written as $f^1 = t_H^1 + g^1$ and $f^2 = t_H^2 + g^2$ for suitable functions g^1 and g^2. Substituting f^1 and f^2 into the first order conditions for utility maximization and defining

$$x' = \exp\left(f^1\right)^{-\gamma(1+\beta_1(1-\rho))} \exp\left(f^2\right)^{-\gamma(1+\beta_2)} \tag{A.19}$$

and

$$y' = \exp\left(f^1\right)^{-\gamma(1+\beta_1)} \exp\left(f^2\right)^{-\gamma(1+\beta_2(1-\rho))} \tag{A.20}$$

we find that the functions g^1 and g^2 must also satisfy the conditions

$$\frac{x'}{y'} = \frac{x}{y}\exp\left(g^1\right)^{\gamma\rho\beta_1}\exp\left(g^2\right)^{\gamma\rho\beta_2)} = \frac{x}{y} \tag{A.21}$$

and

$$x'y' = xy\exp\left(g^1\right)^{-2\gamma(1+\beta_1)+\gamma\rho\beta_1}\exp\left(g^2\right)^{-2\gamma(1+\beta_2)+\gamma\rho\beta_2} = xy. \tag{A.22}$$

Clearly, the only functions that satisfy these constraints are $g^1 = g^2 = 0$. □

Proof of Proposition 3. For the second part we recognize that $|t_S^i|\phi = |t_H^i|$ for $i = 1, 2$ (by Lemma 1) so the equilibrium condition becomes

$$\frac{\rho}{2}\frac{|t_H^1|}{\phi}\beta_1 + \frac{\rho}{2}\frac{|t_H^2|}{\phi}\beta_2 - T - c = 0 \tag{A.23}$$

and rearranging this equation yields (20). For the first part we need to show that T will always be chosen such that the participation constraint is binding. Assume that it is not binding, and as a result, the exchange makes a profit $(1 + \phi)T$. If the exchange keeps T constant at the same time as making the hedgers slightly worse off and the speculators slightly better off, the result is an increase in ϕ and, consequently, an increase in the profits of the exchange. We need to inspect the feasibility of such a strategy. Consider first strategies described by the directions $d\beta_2 = (1-\rho\beta_1)^{-2}\, d\beta_1$ since these ensure that the constraint $\beta_2 > \beta_1/(1-\rho\beta_1)$ is not violated. We find

$$d\phi = \frac{\partial\phi}{\partial\beta_1}\, d\beta_1 + \frac{\partial\phi}{\partial\beta_2}\, d\beta_2 = \left(\frac{\partial\phi}{\partial\beta_1} + \frac{\partial\phi}{\partial\beta_2}(1+\rho\beta_1)^{-2}\right) d\beta_1 \tag{A.24}$$

and it suffices to demonstrate that the expression inside the bracket is not zero. Working out the partials we find (using the transformation of variables defined in (26) and (27))

$$\frac{\partial\phi}{\partial\beta_1} = \frac{\rho\left(1-\frac{\rho}{2}\right)\kappa}{T+c}\left(\frac{1+\ln\delta^{\frac{1}{2\gamma}}}{\beta_1} + \frac{\left(1-\frac{\rho(1-\rho)}{2}\right)\kappa\left(1+\ln\delta^{\frac{1}{2\gamma}}\right)}{\beta_1\beta_2} + \frac{\rho\beta_1\beta_2+\rho\beta_2^2}{2\gamma\delta(\beta_2-\beta_1-\rho\beta_1\beta_2)^2}\right) \tag{A.25}$$

and

$$\frac{\partial\phi}{\partial\beta_2} = \frac{\rho\left(1-\frac{\rho}{2}\right)\kappa}{T+c}\left(\frac{1+\ln\delta^{\frac{1}{2\gamma}}}{\beta_1} - \frac{\left(1-\frac{\rho(1-\rho)}{2}\right)\kappa\left(1+\ln\delta^{\frac{1}{2\gamma}}\right)}{\beta_1\beta_2} - \frac{\rho\beta_1\beta_2+\rho\beta_1^2}{2\gamma\delta(\beta_2-\beta_1-\rho\beta_1\beta_2)^2}\right). \tag{A.26}$$

Clearly, both are non-zero and

$$\frac{\partial\phi}{\partial\beta_1} \neq \frac{\partial\phi}{\partial\beta_2}(1-\rho\beta_1)^2.$$

We can do a similar analysis for the constraint that

$$\beta_2 > \frac{1-\rho(1-\rho)/2}{1-\rho(1+\rho)/2}\beta_1$$

which yields the same result. The proposition follows. □

Proof of Proposition 4. The relationship defining the optimal commission fee T is

$$Eu(\widetilde{w}_0) = Eu(\widetilde{w}_H) \tag{A.27}$$

where $\widetilde{w}_0$ is a $+1/-1$ bivariate random variable and $\widetilde{w}_H$ is given by the table above. The left-hand side equals $-\cosh(\gamma)$ and the right-hand side equals $-\exp(\gamma T)E\exp(\tilde{A})$. The function $\tilde{A}$ is obtained by multiplying the state-contingent wealth in the table above (ignoring the term $-T$) by $-\gamma$. The optimal commission fee is, therefore, obtained by taking logs on both sides in the above relationship, i.e.

$$T = \frac{1}{\gamma}\ln\cosh(\gamma) - \frac{1}{\gamma}\ln E\exp\left(\tilde{A}\right) \tag{A.28}$$

and the proof is complete. □

Proof of Proposition 5. Let Π denote the objective function in (25), and denote by C the ratio

$$C = \frac{\ln\left(\frac{\cosh(\gamma)}{\frac{1}{4}\sum_i \cosh(i)-\rho\sinh(i)}\right)^{\frac{1}{\gamma}}}{c+\ln\left(\frac{\cosh(\gamma)}{\frac{1}{4}\sum_i \cosh(i)-\rho\sinh(i)}\right)^{\frac{1}{\gamma}}}. \tag{A.29}$$

The partials $\partial\Pi/\partial\delta$ and $\partial\Pi/\partial\kappa$ are given by

$$\frac{\partial \Pi}{\partial \delta} = C\rho\left(1-\frac{\rho}{2}\right)\frac{\kappa}{2\gamma\delta} - \frac{\sum_i(\sinh(i)-\rho\cosh(i))\frac{\partial i}{\partial \delta}}{\gamma\sum_i \cosh(i)-\rho\sinh(i)}\left(1+\frac{\rho\left(1-\frac{\rho}{2}\right)\left(1+\ln\delta^{\frac{1}{2\gamma}}\right)c\kappa}{\gamma\left(c+\ln\left(\frac{\cosh(\gamma)}{\frac{1}{4}\sum_i\cosh(i)-\rho\sinh(i)}\right)^{\frac{1}{\gamma}}\right)^2}\right) \tag{A.30}$$

and

$$\frac{\partial \Pi}{\partial \kappa} = C\rho\left(1-\frac{\rho}{2}\right)\left(1+\ln\delta^{\frac{1}{2\gamma}}\right) - \frac{\sum_i(\sinh(i)-\rho\cosh(i))\frac{\partial i}{\partial \kappa}}{\gamma\sum_i \cosh(i)-\rho\sinh(i)}\left(1+\frac{\rho\left(1-\frac{\rho}{2}\right)\left(1+\ln\delta^{\frac{1}{2\gamma}}\right)c\kappa}{\gamma\left(c+\ln\left(\frac{\cosh(\gamma)}{\frac{1}{4}\sum_i\cosh(i)-\rho\sinh(i)}\right)^{\frac{1}{\gamma}}\right)^2}\right). \tag{A.31}$$

At the corner point $\delta = 1$ and $\kappa = 0$ it can easily be confirmed that the second term in (A.30) vanishes, so

$$\frac{\partial \Pi}{\partial \delta} = 0 \tag{A.32}$$

since $\kappa = 0$. Similarly, it can also be confirmed that the second term in (A.31) equals $-(\rho/\gamma)\rho(1-\rho/2)$, so

$$\frac{\partial \Pi}{\partial \kappa} = \rho\left(1-\frac{\rho}{2}\right)\left(C-\frac{\rho}{\gamma}\right). \tag{A.33}$$

This expression is positive if and only if

$$c < \frac{\ln\cosh(\gamma)}{\rho} \quad \text{and} \quad \gamma > \frac{\ln(\cosh(\gamma))}{\ln(\cosh(\gamma))-\rho c}.$$

It suffices (since the feasibility space is open) to demonstrate existence by evaluating (A.30) and (A.31) at the upper bound, which is either at the limit where $\delta \to \infty$ or where $\kappa \to \infty$ (note that the partials $\partial\delta/\partial\beta_i$ and $\partial\kappa/\partial\beta_i$ are both non-zero everywhere in the interior). The main problem evaluating these expressions is linked to the terms

$$\frac{\sum_i(\sinh(i)-\rho\cosh(i))\frac{\partial i}{\partial j}}{\gamma\sum_i \cosh(i)-\rho\sinh(i)},$$

where $j = \delta, \kappa$. We can, however, rewrite these terms in the following way:

$$\frac{\sum_i(\sinh(i)-\rho\cosh(i))\frac{\partial i}{\partial \delta}}{\gamma\sum_i \cosh(i)-\rho\sinh(i)} = \frac{\left(1-\delta^{-\frac{1}{\gamma}}\right)(K^1+K^2)+\left(1+\delta^{-\frac{1}{\gamma}}\right)\kappa\rho\left(\left(1+\frac{\rho(1-\rho)}{2}\right)L^1+\left(1-\frac{\rho(3-\rho)}{2}\right)L^2\right)}{2\delta\gamma^2\left(1+\delta^{-\frac{1}{\gamma}}\right)(K^1+K^2)+\left(1-\delta^{-\frac{1}{\gamma}}\right)(L^1+L^2)} \tag{A.34}$$

and

$$\frac{\sum_i (\sinh(i) - \rho\cosh(i))\frac{\partial i}{\partial \kappa}}{\gamma\sum_i \cosh(i) - \rho\sinh(i)} = \frac{\delta^{\frac{1}{2\gamma}}(1+\delta^{-\frac{1}{\gamma}})(1+\ln\delta^{\frac{1}{2\gamma}})\rho((1+\frac{\rho(1-\rho)}{2})L^1 + (1-\frac{\rho(3-\rho)}{2})L^2)}{2\delta\gamma^2(1+\delta^{-\frac{1}{\gamma}})(K^1+K^2) + (1-\delta^{-\frac{1}{\gamma}})(L^1+L^2)}, \tag{A.35}$$

where

$$K^1 = \frac{1+\exp(-2\kappa\rho(1+\frac{\rho(1-\rho)}{2})(1+\ln\delta^{\frac{1}{2\gamma}}))}{2} + \rho\frac{1-\exp(-2\kappa\rho(1+\frac{\rho(1-\rho)}{2})(1+\ln\delta^{\frac{1}{2\gamma}}))}{2},$$

$$K^2 = \exp\left(-\kappa\rho\frac{4\rho(1-\rho)}{2}(1+\ln\delta^{\frac{1}{2\gamma}})\right)\left(\frac{1+\exp(-2\kappa\rho(1+\frac{\rho(1-\rho)}{2})(1+\ln\delta^{\frac{1}{2\gamma}}))}{2} + \rho\frac{1-\exp(-2\kappa\rho(1+\frac{\rho(1-\rho)}{2})(1+\ln\delta^{\frac{1}{2\gamma}}))}{2}\right),$$

$$L^1 = \frac{1-\exp(-2\kappa\rho(1+\frac{\rho(1-\rho)}{2})(1+\ln\delta^{\frac{1}{2\gamma}}))}{2} + \rho\frac{1+\exp(-2\kappa\rho(1+\frac{\rho(1-\rho)}{2})(1+\ln\delta^{\frac{1}{2\gamma}}))}{2},$$

$$L^2 = \exp\left(-\kappa\rho\frac{4\rho(1-\rho)}{2}(1+\ln\delta^{\frac{1}{2\gamma}})\right)\left(\frac{1-\exp(-2\kappa\rho(1+\frac{\rho(1-\rho)}{2})(1+\ln\delta^{\frac{1}{2\gamma}}))}{2} + \rho\frac{1+\exp(-2\kappa\rho(1+\frac{\rho(1-\rho)}{2})(1+\ln\delta^{\frac{1}{2\gamma}}))}{2}\right).$$

The limits of these terms are, therefore, $\lim_{\delta,\kappa\to\infty} K^1, L^1 = 1/2$ and $\lim_{\delta,\kappa\to\infty} K^2, L^2 = 0$. Using this information in evaluating (A.30) and (A.31) we find

$$\lim_{\delta\to\infty}\frac{\partial\Pi}{\partial\delta} = 0 \tag{A.36}$$

and

$$\lim_{\delta\to\infty}\frac{\partial\Pi}{\partial\kappa} = \begin{cases} -\infty & \text{if } \gamma \leqslant \frac{1}{2}, \\ 0 & \text{otherwise} \end{cases} \tag{A.37}$$

for the bound defined by $\delta\to\infty$, and

$$\lim_{\kappa\to\infty}\frac{\partial\Pi}{\partial\delta}, \frac{\partial\Pi}{\partial\kappa} = -\infty \tag{A.38}$$

for the bound defined by $\kappa\to\infty$. Ignoring the case where $\gamma\leqslant 1/2$, since in this case the corner solution is optimal, we can use the following argument to demonstrate existence. It is easy to confirm that the boundary point described by δ is in effect only if

$$\beta_1 > \frac{\rho}{1-\rho(1-\rho)/2}$$

(using the rule that

$$\max\left(\frac{\beta_1}{1-\rho\beta_1}, \frac{1-\rho(1-\rho)/2}{1-\rho(1+\rho)/2}\right) = \frac{1}{1-\rho\beta_1}).$$

However, along the boundary points the exchange is indifferent for all choices of β_1 and β_2, including those for which

$$\beta_1 \leqslant \frac{\rho}{1-\rho(1-\rho)/2},$$

in which case the exchange always prefers an interior point. □

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